Dynamic Buckling of Ring Stiffened Cylindrical Shells

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The dynamic stability of ring stiffened cylindrical shells under axial and lateral step loads is investigated. The effects of initial geometric imperfection and the eccentricity of the stiffeners on the dynamic buckling loads of the shell are determined. Studies show that ring-stiffened cylinders are imperfection-sensitive and can buckle at a load considerably smaller than that predicted for perfect shells. The study also demonstrated that the essential mechanism for the imperfection sensitivity in the dynamic range is the so-called "column" asymmetric mode with $m=1$ and $n>>1$.

Introduction

THE compressive stability of stiffened cylindrical shells has interested researchers during the past decade on two major points. First, available test data seemed to indicate that the stiffened shells (including ring-stiffened, stringer-stiffened, or combined case) failed at loads very close to the ones predicted by a classical linear theory. This implied, at least for some ranges of the stiffener parameters, that the stiffened-shells may be less imperfection-sensitive than the unstiffened cylindrical shells where agreement with classical linear theory was an exception to the rule. The second feature of the stiffened shell problem is the influence of stiffener eccentricity on the buckling load. The location of the stiffener with respect to the shell center-line seemed to increase or decrease the buckling load considerably, and in some cases, the buckling modes were also altered.

Encouraged by the test results, an extensive body of literature was developed in the sixties dealing with the classical buckling of (perfect) stiffened shells under a variety of loads; a sampling of which may be seen in Refs. 1-3. However, Hutchinson and Amazigo[4] were the first to sound a note of caution and suggest that the so-called imperfection insensitivity of stiffened shells is not necessarily true for all ranges of the stiffened shell parameters and showed that considerable reduction in the predicted loads are possible if initial imperfections are taken into account.

While all this was addressed to static stability, the dynamic stability of stiffened cylindrical shells under step loading has received very little attention. Reference 7, which deals with the dynamic buckling of unstiffened cylindrical shells under axial step loading, discusses some of the earlier work of Volmir, Nash, and others, where they had considered simplified and somewhat unsatisfactory models. In Refs. 5 and 6, which form part of the current study, the case of axially stiffened cylinders under axial step loads are discussed.

It is to be noted that most of the work on the subject has been analytical and very little experimental work has been reported. A very recent work on the subject[8] concerns itself with unstiffened cylindrical shells. In this paper, we are concerned with the dynamic buckling of imperfect ring-stiffened shells under axial and lateral step loads separately and in combination.

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Governing Equations

The basic equations pertinent to the present problem are similar to those derived in Ref. 6 for axially stiffened shells under step loads. However, to make the paper self contained, a brief resume is included. In the formulation of these equations the following assumptions are made: a) Donnell's nonlinear shell theory is applicable; b) the stiffeners are close enough so that they can be smeared out in calculating the effective rigidities; c) the stiffeners are essentially beam-like elements carrying no shear; d) the entire shell including the stiffeners is activated at buckling, that is, local instability failures are ruled out; and e) longitudinal and tangential inertia terms are of lower order of importance compared to normal inertia.

Figure 1 shows the cylindrical shell with the stiffener geometry and the coordinate system. Figure 1 also shows the loading in the form of a step-function, where the load is applied suddenly at $t=0$ and kept on indefinitely. With the normal to the shell surface taken as positive inward, positive (negative) values of $\bar{E}_r$ indicate inside (outside) location of the stiffeners with respect to the shell.

The unstiffened shell is characterized by three parameters: L, R, and H (length, radius, and the thickness of shell), which can be combined into the so-called Batdorf parameter $Z=L^2/RH$, the ranges of whose values are used to describe a "short" or a "moderate-length" cylinder. For practical thin shells Z is in the range 10^2-10^4.

The effective increases in the area of section, the section area moment, and the section torsional stiffness of the shell due to the presence of stiffeners are best characterized by nondimensional parameters α_r, η_r, γ_r. They are respectively defined as follows:

$$\alpha_r \text{ (axial stiffness parameter)} = ((1-\nu^2)/h)\ (A_r/b_r)$$

$$\eta_r \text{ (flexural stiffness parameters)} = (E/D)\ (I_r/b_r)$$

$$\gamma_r \text{ (torsional stiffness parameter)} = (G/D\ (1-\nu))\ (J_r/b_r)$$

Where E is the Young's modulus, G the modulus of rigidity, ν Poisson's ratio, h thickness of stiffeners, $D=EH^3/12(1-\nu^2)$, b_r the spacing between ring stiffeners, and A_r, I_r, J_r are, respectively, the cross-sectional area, moment of inertia about shell middle surface and torsional constant of ring stiffeners.

We introduce the following nondimensional quantities

$$u,v,w,\bar{w},x,y,\bar{e}_r=[U,V,W,\bar{W},X,Y,\bar{E}_r]/R \tag{1a}$$

$$n_x,n_y,n_{xy},\bar{n}_x=[N_x,N_y,N_{xy},\bar{N}_x]/B \tag{1b}$$

$$m_x,m_y,m_{xy},m_{yx}=R[M_x,M_y,M_{xy},M_{yx}]/D \tag{1c}$$

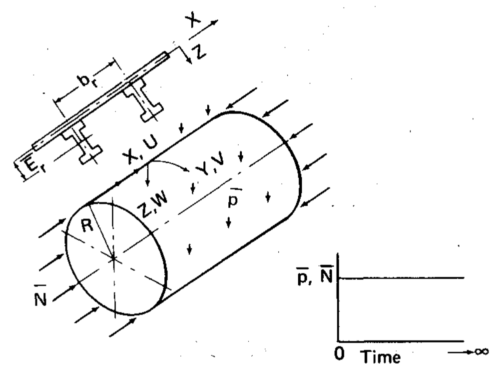

Fig. 1 Shell geometry and loading.

where U, V, W are the middle surface displacements of the shell, $\bar{W}$ initial imperfection of shell middle surface, X, Y the axial and circumferential coordinates, $\bar{E}_r$ the ring stiffener eccentricity, N_x, N_y, N_{xy} the stress resultants; M_x, M_y, M_{xy}, M_{yx}, are the moments, $\bar{N}_x$ is the applied axial load, and $B=EH/12(1-\nu^2)$.

It is the characteristic of the present stiffened-shell formulation that the stiffeners give rise to coupling between the membrane stress resultant n_y and the twist k_y, and one between bending resultant m_y and direct strain ϵ_y^0. Also $m_{xy} \neq m_{yx}$. This is seen in Eq. (2).

$$\begin{bmatrix} n_x \\ n_y \\ n_{xy} \\ m_x \\ m_y \\ m_{xy} \\ m_{yx} \end{bmatrix} = \left[\begin{array}{ccc|cccc} 1 & \nu & 0 & 0 & 0 & 0 & 0 \\ \nu & (1+\alpha_r) & 0 & 0 & -\bar{e}_r R\alpha_r & 0 & 0 \\ 0 & 0 & (1-\nu) & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & \nu & 0 & 0 \\ 0 & -\bar{e}_r\beta_r & 0 & \nu & (1+\eta_r) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (1-\nu)(1+\gamma_r) \end{array}\right] \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \epsilon_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \kappa_{yx} \end{bmatrix} \quad (2)$$

Equations of Motion

The equations of motion pertinent to the problem are the familiar Donnell-Karman type of coupled PDE, modified to include the effect of stiffening and are

$$L_1(f) - L_2(w) - \bar{A}_r(w^2,_{xy} - w,_{xx}w,_{yy} - w,_{xx} + 2w,_{xy}\bar{w},_{xy}$$
$$- w,_{xx}\bar{w},_{yy} - w,_{yy}\bar{w},_{xx}) \quad (3)$$

$$-(1/12z^2)L_3(w) - (1/\bar{A}_r)[L_2(w)]$$
$$+f,_{xx}(w^*,_{yy}+1) + f,_{yy}w^*,_{xx} - 2f,_{xy}w^*,_{xy} = (\partial^2 w/\partial\tau^2) \quad (4)$$

where f is a stress function, and commas denote partial differentiation with respect to the subscripts following the comma and $w^* = w + \bar{w}$. The linear operators, L_1, L_2, and L_4 are given by

$$L_1(\,) = (\,),_{xxxx} + 2(1 + \frac{\alpha_r}{1-\nu}(\,),_{xxyy} + (1+\alpha_r)(\,),_{yyy} \quad (5a)$$

$$L_2(\,) = -e_r\alpha_r(\,),_{xxyy} + \nu e_r\alpha_r(\,),_{yyyy} \quad (5b)$$

$$L_3(\,) = (\,),_{xxxx} + 2(1 - \frac{\alpha_r}{1+\nu})(\,),_{xxyy} + (1+\eta_r)(\,),_{yyyy} \quad (5c)$$

$$L_4(\,) = (e_r\alpha_r)^2(\,),_{yyyy} \quad (5d)$$

and $\bar{A}_r = 1+\alpha_r-\nu^2$; $\tau = (t/R)\,[E\bar{A}_r/\rho(1-\nu^2)]^{1/2}$ where t is the time, and ρ is the density of shell material. These equations are obvious generalizations of the corresponding isotropic equations, and the latter are obtained by setting all stiffening terms equal to zero.

Following the same approach as employed in Refs. 6 and 7, the normal displacement of the shell is approximated by the following 4-parameter expression which represents the diamond-shaped buckles familiar in the static stability analyses. This model, which is strictly neither simply-supported not clamped, has been found applicable for shells with moderate length range ($z=10^2$ to 10^4) and includes both axisymmetric and asymmetric modes, together with a breathing mode.

$$w = (H/R)\;[a_1(\tau)\cos(m\pi R/L)x\cos ny$$
$$+ a_2(\tau)\cos(2m\pi R/L)x + a_3(\tau)\cos ny + a_4(\tau)] \quad (6)$$

The initial geometric imperfection $\bar{w}$ is taken as

$$\zeta\bar{w} = (H/R)\,[d_1\cos(m\pi R/L)x\cos ny$$
$$+ d_2\cos(2m\pi R/L)x] \quad (7)$$

Expressions similar to these have been used by Tamura[8] in his investigation of dynamic stability of unstiffened shells. In Eqs. (6) and (7), m and n represent the wave numbers in the axial and circumferential directions, respectively.

The procedure is now straightforward. We substitute Eqs. (6) and (7) in the compatibility equation, Eq. (3); we solve for f and obtain a particular solution. This solution for f is substituted in Eq. (4) to obtain a single equation in w. This is solved, then, by using a Galerkin-type procedure along with the condition of single-valuedness of the displacement along the circumference of the shell to yield 4 nonlinear ordinary differential equations for the normal coordinates a_1 to a_4. These steps are exactly similar to those followed in Ref. 6 and will not be repeated here. The resulting equations may be written in the following compact form

$$\ddot{a}_1 + a_1[\hat{n}_{st} - (1+k\beta^2)]\hat{n} + f_1(a_1,a_2,a_3,a_4,d_1,d_2) = 0$$
$$k \leq 1 \quad (8a)$$

$$\ddot{a}_1 + a_1[\hat{p}_{st} - (1/k+\beta^2)]\hat{p} + f_1(a_1,a_2,a_3,a_4,d_1,d_2) = 0$$
$$k > 1 \quad (8b)$$

$$\ddot{a}_i + a_i g_i(\hat{n}) + f_i(a_1,a_2,a_3,a_4,d_1,d_2) = 0$$
$$i = 2,3,4 \quad (9)$$

In Eqs. (8) or (9), f_i are nonlinear functions of $(a_1, a_2, ..., d_2)$ containing powers and products whose form is similar to those in Eqs. (18-21) of Ref. 6. $\hat{n}$ and $\hat{p}$ are further dimensionless applied axial load and lateral pressure load respectively, related to actual loads n_x and p through

$$\hat{n} = (\bar{N}_x/B) \quad (Z/A_r) \text{ and } \hat{p} = (pRZ/B\bar{A}_r), k = (\hat{p}/\hat{n})$$

$k=0$ for pure axial loading and $k=\infty$ for pure lateral loading. Furthermore, $\hat{n}_{st}$, $\hat{p}_{st}$ stand for the expressions which will yield the static linear (classical) value in the limiting case. Thus, for example, if only linear static case is considered. f_l and $\ddot{a}_l$ are necessarily equal to zero; then with only axial loading, i.e. $k=0$, we have from Eq. (8a)

$$\hat{n} = \hat{n}_{st}$$

Where $\hat{n}_{st}$[6] comprises geometric and stiffness parameters of the shell together with m and n, the wave numbers; the minimized value of this yields the linear static buckling coefficient.

Input Parameters

The input parameters include the stiffening parameters α_r, η_r; the eccentricity parameter e_r; Batdorf parameter Z; imperfection parameters d_l and d_2; load parameters n, k; and the wave numbers m and n. The range of values for the ring stiffness parameters has been chosen to conform to light, medium, and heavy stiffenings.

Of the remaining structural parameters, Z has been taken as 1000 and the imperfection parameter d_2 set to zero; d_l has been varied between (0, 1). The loads $\hat{n}$ and $\hat{p}$ are assumed to be applied suddenly and kept thereon indefinitely; i.e., in the form of step functions. Since for every integer value of m and n we can associate a critical load and the buckling load is the lowest of these, we have a wide range of m and n values.

Buckling Load: Theoretical Considerations

For a given set of input parameters, Eqs. (8) and (9) can be integrated numerically, with appropriate initial conditions, to yield $a_l - a_4$ as functions of τ. A Runge-Kutta scheme was used to integrate the system of equations, and the convergence of the solution was verified by altering the step size.

The buckling criterion used here is similar to the one derived by Budiansky and Roth,[9] where a jump in the peak amplitude of the deflection is associated with a critical load. The jump to be noted here is that of the unit end shortening defined through

$$\delta = -\int_0^l u_{,x} \mathrm{d}x \tag{10}$$

which can be written after substituting for u in terms of f and w as

$$\delta Z = (1+\alpha_r)\hat{n} + \nu\hat{p} + [(m\pi)^2/8Z] \; [a_l^2 + 8a_2^2 + 2a_l d_l + 16a_2 d_2] \tag{11}$$

Modal Considerations

We have already remarked that in obtaining the buckling load we have to try a large number of m and n values to insure that we obtain the lowest critical values. This can be extremely time consuming even with high-speed digital computers.

However, this process may be made quite manageable by making use of some unique features of stiffened shells. Linear static buckling analysis of stiffened shells[1,2] show that classical linear buckling loads for a stiffened shell have associated with them unique buckling modes. By varying the stiffness parameters one can establish[10,11] a wide range of these parameters for which the associated buckling modes have constant and convenient values, such as $m=1$ or $n=0$, depending upon the loading. Now we wish to establish the relevance of linear static value to the present dynamic problem by considering the limiting case of perfect cylindrical shell (i.e., with $d_l = d_2 = 0$).

Perfect Cylinder Solution

Without loss of generality we consider the pure axial loading $\hat{n}$ and after making use of Eqs. (8, 9, and 11), we obtain an equation of the following form as the undamped dynamical equation of perfect stiffened shells

$$\ddot{\delta} - \delta(\hat{n} - G) + f\,(\text{nonlinear terms in } \delta, \text{ stiffening parameters}) = 0 \tag{12}$$

where $G = G(m, n, Z, \alpha_r, e_r, \eta_r)$ corresponds to a critical load. For a cylinder of given stiffness parameters and Z, $G_{\min}$ with respect to m,n gives the linear static (classical) buckling load of the stiffened cylinder. It is important to note that the corresponding m,n are unique.

Although Eq. (12) is nonlinear, it is apparent that the coefficient of the linear term alone governs the boundedness of the solution which is possibly only for $\hat{n} \geq G$. The nonlinearity only distorts the amplitude without affecting the basic stability. The minimum value for which the solution is bounded is obviously given by $\hat{n} = G_{\min}$.

Thus, the static *linear value provides the buckling solution for the dynamic stability of a perfect (stiffened) cylinder.* As a corollary, by letting all stiffening parameters go to zero, $G_{\min}$ would correspond to the static linear (classical) value of isotropic cylinders. However, in this case the m,n values will not be unique. This fact that for a perfect cylinder the dynamic buckling solution is the static (classical) solution has been noted by Roth and Klosner.[7]

Having thus selected the relationship of linear static value to the dynamic stability problem, it is only reasonable to assume that the mode shapes of the linear problem are relevant in the nonlinear problem and it is worthwhile determining the linear static buckling modes of the given stiffened cylinder as a preliminary step to the dynamic problem; the linear static problem is easy to formulate and takes very little computer time to obtain the mode numbers for a wide variety of stiffening and shell geometries as well as loading. The mode numbers can then be profitably used for the nonlinear dynamic problem; this feature was already exploited (and dealt with at some length) in Ref. 6.

Method of Procedure

The general method of procedure for calculating the dynamic buckling load for the given imperfection of the stiffened cylinder is illustrated in Figs. 2a and 2b which are reproduced from Ref. 6. For the given stiffener geometry a preliminary linear analysis is performed to determine the linear buckling load as well as the modes. Using these modes as starting input values in Eqs. (8) and (9) and assuming initial conditions of zero displacements and velocities i.e., $a_i = \dot{a}_i = 0$ $(i=1,2...4)$, a_i are evaluated using a Runge-Kutta scheme. From Eq. (11), δ, the end shortening is obtained as a function of τ the nondimensional time parameter. Fig. 2a shows such a sample output. τ values of more than several hundreds, corresponding to about 10 msec in real time were used to check the oscillatory characters and to make sure that later peaks were no higher than the ones encountered in the first two cycles.

In Fig. 2b, a plot of $\delta_{\max}$ corresponding to the peaks in Fig. 2a are plotted as a function of the load for a given set of m,n. It is noted that a clearly defined jump occurs at the critical load. The minimum of all critical loads for all m,n values corresponds to the buckling load. It was found in Ref. 6 that

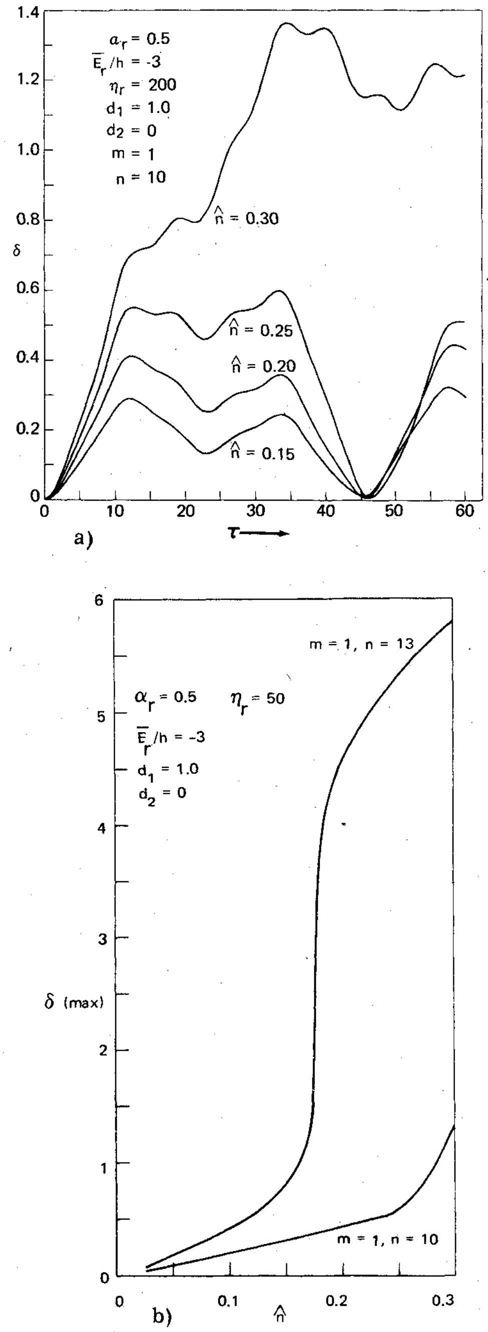

Fig. 2 Variation of end shortening δ and variation of δ max with $\hat{n}$.

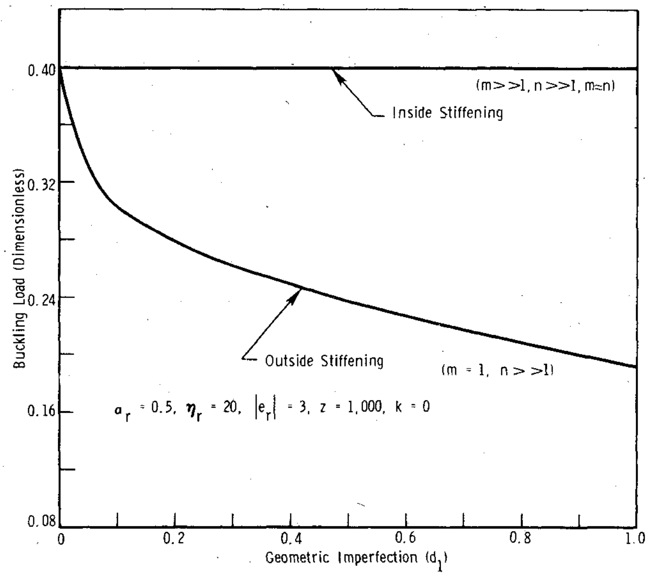

Fig. 3 Imperfection sensitivity of a ring-stiffened cylinder under axial (step) loading.

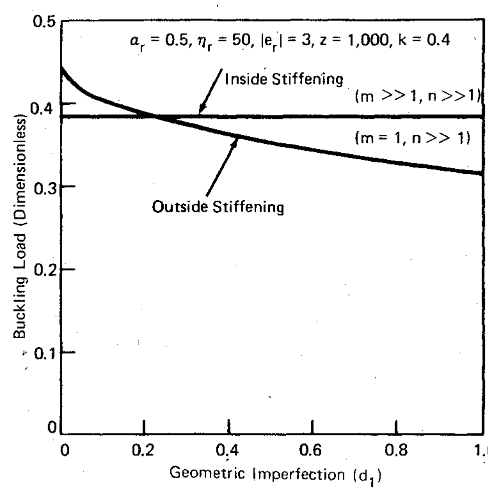

Fig. 4 Imperfection sensitivity of a ring-stiffened cylinder under combined axial and lateral (step) loading.

the m,n values from the linear static buckling analysis indeed turned out to be the minimizing values for the dynamic imperfect case also. This further emphasizes the importance of the preliminary linear static analysis as it a) provides the zero imperfection point in a typical imperfection sensitivity plot; and b) provides the modal numbers (very often the correct values) for input in the imperfection range.

Results and Discussion

Figure 1 shows the typical loading and geometrical parameters of a ring-stiffened shell. The lateral and axial loads are taken as step functions, i.e., the load is applied suddenly at $t=0$ and kept on indefinitely. The classical (static linear) values corresponding to the pure axial or pure lateral pressure cases are referred to 2 buckling loads, $\hat{n}_{st}$ and $\hat{p}_{st}$, respectively. The applied lateral pressure $\hat{p}$ is introduced as a fraction k of the applied axial load $\hat{n}$.

In Figs. 2a and 2b, we give the illustrations for the method of procedure to determine the dynamic buckling load. In Figs. 3-7, we have the results for the (dynamic) imperfection sensitivity of a ring-stiffened cylindrical shell for a particular choice of stiffening parameters under step loading, ranging from a pure axial case $k=0$ to a pure lateral case $k=\infty$.

Figure 3 shows that for a pure axial loading the imperfection sensitivity of external rings is quite marked, similar to the case of external stringers under axial load.[7] However, internal rings show almost no imperfection sensitivity, that is, we fail to find a dynamic buckling load, based on the buckling criterion of a large structural response to a small change in the load, for any combination of m,n which is below the static linear classical buckling load for a given imperfection parameter.

A possible explanation is that the internally ring-stiffened cylinders buckle in an asymmetric mode with large number of axial waves. i.e. $m \gg 1$. This has been observed in the linear studies of stiffened shells.[1,2] When the cylinder buckles with many axial waves, there is almost a "pulse-buckling" effect, discussed by Lindberg[12] and Bolotin[13] for unstiffened cylinders. With such local instability, the overall snap-through type of instability associated with dynamic buckling, fails to take place and hence we fail to find a lower dynamic buckling load. In contrast to this behavior, the external rings (also as external stringers[7]) invariably buckle with an $m=1$ asym-

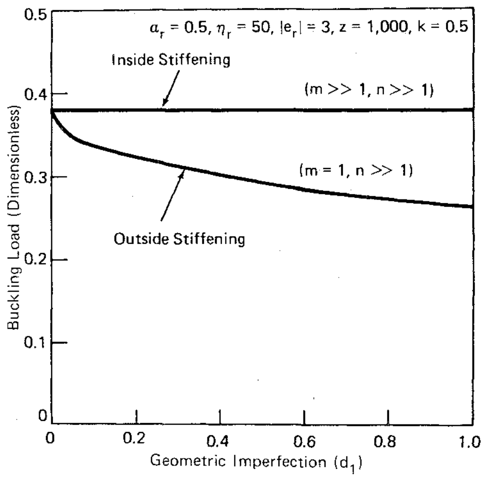

Fig. 5 Imperfection sensitivity of a ring-stiffened cylinder under combined axial and lateral (step) loading.

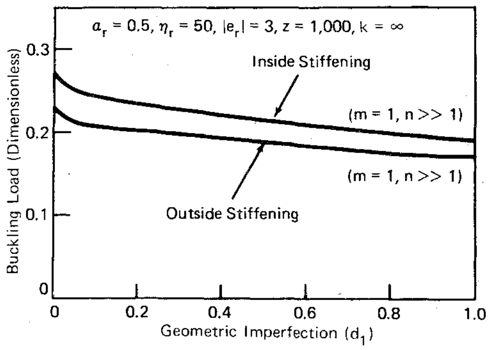

Fig. 6 Imperfection sensitivity of a ring-stiffened cylinder under lateral (step) loading.

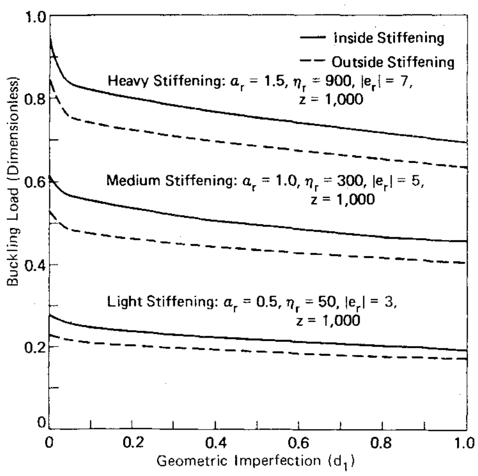

Fig. 7 Imperfection sensitivity of a ring-stiffened cylinder under lateral (step) loading.

metric mode where the overall snap-through is readily achieved. We can conveniently call this a "column" type of nonaxisymmetric mode with $m = 1$, $n \gg 1$.

In Figs. 4 and 5, we have a small addition of external pressure given by $k = 0.4$ and 0.5, respectively. Even with this extra lateral pressure the internal rings are fully effective, i.e. they break up the wave pattern, creating $m \gg 1$ type modes which makes the shell relatively imperfection sensitive under the dynamic (step) loading. The external rings still are "soft" and an $m = 1$, $n \gg 1$. "Column" type mode prevails with a consequent lower dynamic buckling load.

It may be noticed in Fig. 5 that the static linear (classical) value of the external rings are lower than that of internal rings. This reversal of eccentricity is a well-known phenomenon discussed thoroughly in Refs. 1 and 2.

The stage when both external and internal rings become ineffective, and both exhibit "column" mode of $m = 1$, $n \gg 1$, occurs almost immediately after any addition of pressure beyond $k = 0.5$. Also the spread between the internal and external-ring cases for the respective static values remains nearly the same.

In Fig. 6 the extreme case of $k = \infty$ with only external pressure acting is shown; both internal and external-ring cases are imperfection sensitive and the eccentricity effect is fully reversed.

It is important to emphasize here that the critical value of additional pressure load needed to deactivate the rings is necessarily a function of the relative 'heaviness' of the rings. Thus, with heavier rings, the k value would become larger before the rings are rendered ineffective.

In Fig. 7 the dynamic results for pure lateral pressure loading of ring-stiffened shells are presented. Here, even for relatively heavier rings, the 'column' mode of $m = 1$ prevails and both external and internal-ring cases show marked imperfection sensitivity.

Conclusions

In conclusion, we find from the present study, as well as the axially stiffened case,[6] the following important features of the dynamic imperfection-sensitivity problem of stiffened cylindrical shells under step loads:

1) Both axially stiffened and ring-stiffened cylinders are imperfection-sensitive and can buckle at a considerably small fraction of the load predicted for perfect shells.

2) Ring-stiffened shells with only internal rings appear to be imperfection-insensitive under axial step loading. With the addition of even small amounts of lateral pressure, "light" rings will become less effective, and the ring-stiffened shells become imperfection-sensitive.

3) Ring-stiffened shells under pure lateral pressure are imperfection-sensitive for all ranges of ring 'heaviness.'

4) The essential mechanism for the imperfection sensitivity in the dynamic range is the so-called 'column' asymmetric mode $m = 1$, $n \gg 1$.

5) Internal rings essentially break up the wave pattern and make the wavelength conform to the ring spacing, when only axial loading is considered, producing $m \gg 1$ type of modes. In this case, the shell is relatively imperfection-insensitive. Addition of lateral pressure essentially weakens the ring effectiveness and produces eventually the 'column' mode.

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